Modeling and Simulation of Flexible Plate On Spacecraft Based on Kane Dynamics

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Abstract: The spacecraft's on-orbit operation progresses toward a fast maneuvering direction. Such mission requirements require more precise requirements for spacecraft modeling and simulation with flexible attachments. In this paper, for the flexible attachment, the cantilever plate is selected as the research object. The Kane dynamic method is used to analyze the "one-time dynamic model" of the flexible attachment. The simulation results of typical motion conditions are given by using MATLAB. By comparing with the simulation results of "zero-order dynamic model", the conclusion that the Kane method can improve the accuracy of dynamic behavior under fast motion is obtained.

1. Introduction

In the context of engineering technology development, spacecraft is developing in the direction of multi-functional and high-task demand. In this context, the number of flexible attachments on spacecraft is also increasing, and the flexibility is gradually increasing. In this context, modeling and analysis of flexible spacecraft through simulation analysis is particularly important. Improving the accuracy of the model has become a hot topic in many studies.

The Kane dynamics method is a dynamic method proposed by T.R.Kane in the 1960s. It is considered by many researchers to be a method that combines the advantages of both vector mechanics and analytical mechanics. The method is based on the selection of "generalized rate". Based on this, kinematics is described. The generalized principal dynamics and generalized inertial forces are constructed to establish the dynamic equations[1]-[6].

There is a kind of "dynamic stiffening" phenomenon in the dynamics problem, which belongs to the category of geometric nonlinearity. The research on this kind of phenomenon is one of the hotspots in the field of dynamics. In recent years, researchers at home and abroad have conducted related research. The entry points for this type of research are not the same, and the applied dynamics methods are not the same. The core purpose is to improve the accuracy of the dynamic model.

In this paper, the cantilever plate dynamics model established by Kane method is used to simulate and compare the “one-time dynamic model” and the “zero-order dynamic model” obtained by the traditional method. This shows that the "one-time dynamics model" can really improve the modeling accuracy and the Kane method can indeed carry out more detailed modeling analysis for spacecraft accessories.

2. Kane method dynamics model

2.1 Kane dynamic equation

The Kane dynamic equation corresponding to the cantilever plate under arbitrary motion is obtained by formula:

Similar to the dynamic equation of the beam model, K_{ki}^{dX} and other terms represent the dynamic stiffness term, and its contribution to the system is related to the speed and angular velocity of the system. If you ignore this idea, you may get a zero-order approximate kinetic model, which may lead to a completely wrong conclusion when the speed is high.

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$$
\begin{aligned}
&\sum_{j=1}^{\mu_1} W_{ij}^{11}\ddot{q}_{1j} - 2w_3\sum_{j=1}^{\mu_2} W_{ij}^{12}\dot{q}_{2j} + 2w_2\sum_{j=1}^{\mu_3} W_{ij}^{13}\dot{q}_{3j} - (w_2^2 + w_3^2)\sum_{j=1}^{\mu_1} W_{ij}^{11} q_{1j} \\
&+(w_1 w_2 - \dot{w}_3)\sum_{j=1}^{\mu_2} W_{ij}^{12} q_{2j} + (w_1 w_3 + \dot{w}_2)\sum_{j=1}^{\mu_3} W_{ij}^{13} q_{3j} + \sum_{j=1}^{\mu_1} K_{ij}^{S1} q_{1j} \\
&+\sum_{j=1}^{\mu_2} K_{ij}^{S2} q_{2j} = (w_2^2 + w_3^2) X_{1i} - (w_1 w_2 - \dot{w}_3) Y_{1i} - (\dot{v}_1 + w_2 v_3 - w_3 v_2) Z_{1i}
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
&\sum_{i=1}^{\mu_2} W_{ji}^{22}\ddot{q}_{2i} + 2w_3\sum_{i=1}^{\mu_1} W_{ji}^{21}\dot{q}_{1i} - 2w_1\sum_{k=1}^{\mu_3} W_{jk}^{23}\dot{q}_{3k} + (w_1 w_2 + \dot{w}_3)\sum_{i=1}^{\mu_1} W_{ji}^{21} q_{1i} \\
&+(w_2 w_3 - \dot{w}_1)\sum_{k=1}^{\mu_3} W_{jk}^{23} q_{3k} - (w_1^2 + w_3^2)\sum_{k=1}^{\mu_2} W_{jk}^{22} q_{2k} + \sum_{k=1}^{\mu_2} K_{jk}^{S3} q_{2k} \\
&+\sum_{k=1}^{\mu_1} K_{jk}^{S4} q_{1k} = -(w_1 w_2 + \dot{w}_3) X_{2j} + (w_1^2 + w_3^2) Y_{2j} - (\dot{v}_2 + w_3 v_1 - w_1 v_2) Z_{2j}
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
&\sum_{i=1}^{\mu_3} W_{ki}^{33}\ddot{q}_{3i} + 2w_1\sum_{i=1}^{\mu_3} W_{ki}^{32}\dot{q}_{2i} - 2w_2\sum_{k=1}^{\mu_1} W_{ki}^{31}\dot{q}_{1i} + (w_2 w_3 + \dot{w}_1)\sum_{i=1}^{\mu_2} W_{ki}^{32} q_{2i} \\
&+(w_1 w_3 + \dot{w}_2)\sum_{i=1}^{\mu_1} W_{ki}^{31} q_{1i} - (w_1^2 + w_2^2)\sum_{k=1}^{\mu_3} W_{ki}^{33} q_{2i} + (w_2^2 + w_3^2)\sum_{k=1}^{\mu_3} Kdx_{ki}^2 q_{3i} \\
&-(\dot{v}_1 + w_2 v_3 - w_3 v_2)\sum_{i=1}^{\mu_3} Kdx_{ki}^0 q_{3i} - (w_1 w_2 - \dot{w}_3)\sum_{i=1}^{\mu_3} Kdx_{ki}^1 q_{3i} + (w_1^2 + w_3^2) \\
&\sum_{i=1}^{\mu_3} Kdy_{ki}^2 q_{3i} - (\dot{v}_2 + w_3 v_1 - w_1 v_3)\sum_{i=1}^{\mu_3} Kdy_{ki}^0 q_{3i} + (w_1 w_2 + \dot{w}_3)\sum_{i=1}^{\mu_3} Kdy_{ki}^1 q_{1i} \\
&= -(w_1 w_3 - \dot{w}_2) X_{3k} - (w_2 w_3 + \dot{w}_1) Y_{3k} - (\dot{v}_3 + w_1 v_2 - w_2 v_1) Z_{3k}
\end{aligned} \tag{3}
$$

2.2 Simplified cantilever plate dynamic equation

In the dynamic simulation analysis of the cantilever plate, the solution of the dynamic equation is still solved by the $Wilson-\theta$ algorithm. Using the method similar to that when analyzing the cantilever beam, the cantilever plate is used for the rotation of the axis to calculate the vibration deformation[7]-[11].As shown in Figure 1. Variable as follows:

$v_1 = v_2 = v_3 = \dot{v}_1 = \dot{v}_2 = \dot{v}_3 = 0; w_3 = \dot{\theta}, w_1 = w_2 = \dot{w}_1 = \dot{w}_2 = 0; \dot{w}_3 = \ddot{\theta}.$

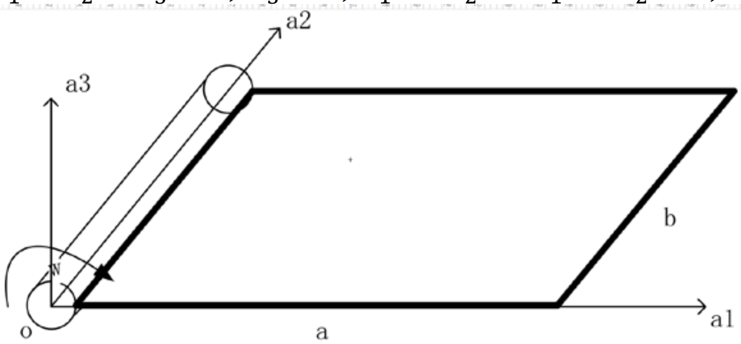

Figure 1 Schematic diagram of cantilever plate rotation analysis

The simplified equation under the assumption is obtained by the above assumptions:

$$
\begin{aligned}
&W_{11}^{11}\ddot{q}_1 + 2\dot{\theta}W_{11}^{13}\dot{q}_3 + (K_{11}^{S1} - \dot{\theta}^2 W_{11}^{11}) q_1 + \ddot{\theta}W_{11}^{13} q_3 = \dot{\theta}^2 X_{11} \\
&W_{11}^{33}\ddot{q}_3 - 2\dot{\theta}W_{11}^{31}\dot{q}_1 - \dot{\theta}^2 W_{11}^{31} q_1 + (-\dot{\theta}^2 W_{11}^{33} + K_{11}^{B} + \dot{\theta}^2 Kdx_{11}^2) q_3 = \dot{\theta}^2 X_{11}
\end{aligned} \tag{4}
$$

It can be seen from the above formula that the Kdx_{ij}^l term represents the dynamic stiffness supplement. If the nonlinear coupling term is ignored at the beginning of the derivation, the inaccurate equations that do not account for the dynamic stiffening term are obtained.

3. Simulation and analysis

3.1 Model parameter description

When analyzing the cantilever plate model, it is also assumed that the fixed end of the cantilever beam is connected to the fixed rotating shaft, and the simplified model is not considered, as shown in Figure 2.

The relevant parameters of the cantilever beam model [18]-[19] are shown in Table 1.

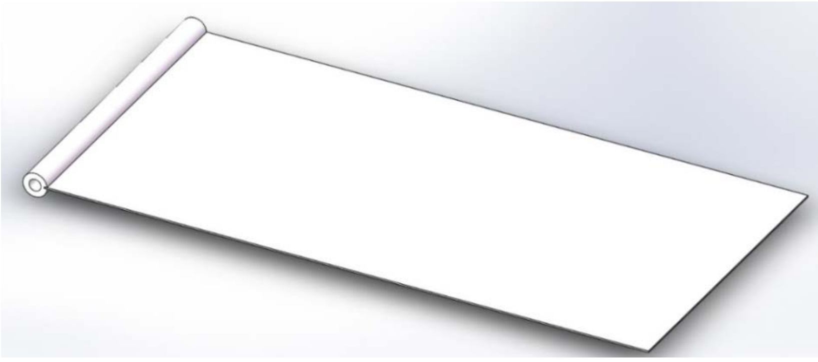

Figure 2 Cantilever plate model

Table 1 Cantilever plate related parameters

Physical property	Density /($kg \cdot m^{-2}$)	Length/m	Width/ m	thickness / m
Value	7.5	1	0.5	0.0025
Physical property	Young's modulus /($N \cdot m^{-2}$)	Shear modulus /($N \cdot m^{-2}$)	Poisson's ratio	
Value	7×10^{10}	2.7×10^{10}	0.3	

3.2 Simulation analysis of the model

The rotation function of the fixed-axis rotation is also a commonly used acceleration rotation equation, which has the characteristics of smoothing of the growth and reaching the steady-state angular velocity after the acceleration time T.

$$\dot{\theta} = \begin{cases} \frac{\Omega}{T}(t - \frac{T}{2\pi}\sin\frac{2\pi t}{T}) & 0 \le t \le T \\ \Omega & t \ge T \end{cases} \tag{5}$$

Where Ω is the steady-state rotational angular velocity and T is the acceleration time, generally taking T=15s.

Through the modal calculation analysis based on ANSYS Workbench, the first-order vibration mode of the cantilever plate is 13.2 rad/s. Therefore, the rotational angular velocities of the simulation analysis are: 10 rad/s, 13.2 rad/s, 15 rad/s.

According to the vibration mode of the finite element calculation analysis, the modal function selection method of the cantilever plate model analysis in Chapter 2 can be used [7]. In this paper, the selection of the modal function is through the selection method in the literature. Selected, the modal function [8] is:

$$\begin{aligned} \phi_{11}(x) &= \sin(\frac{\pi x}{2a}) \\ \phi_{21}(x) &= 1 - \cos(\frac{\pi x}{2a}) \end{aligned} \tag{6}$$

Brought into the equation (4) for the programmatic solution to obtain the simulation analysis results are as follows, the figure shows the comparison between the dynamic justification term and the two cases without considering the dynamic stiffening term.

It can be seen from Figure 3-Figure 5 that in the dynamic modeling, if the influence of the dynamic stiffness term is not considered, the motion displacement tends to diverge when simulating a large range of motion, and when the rotational speed is not high, The trend is the same for both, but the accuracy is still different. When near the first-order natural frequency, the displacement vibration curve of the zero-order model shows a curve similar to resonance.

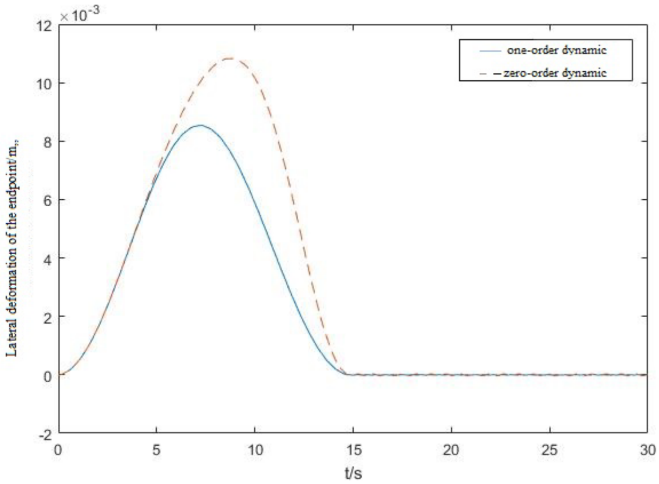

Figure 3 when $\omega = 10rad/s$ plate lateral vibration end displacement

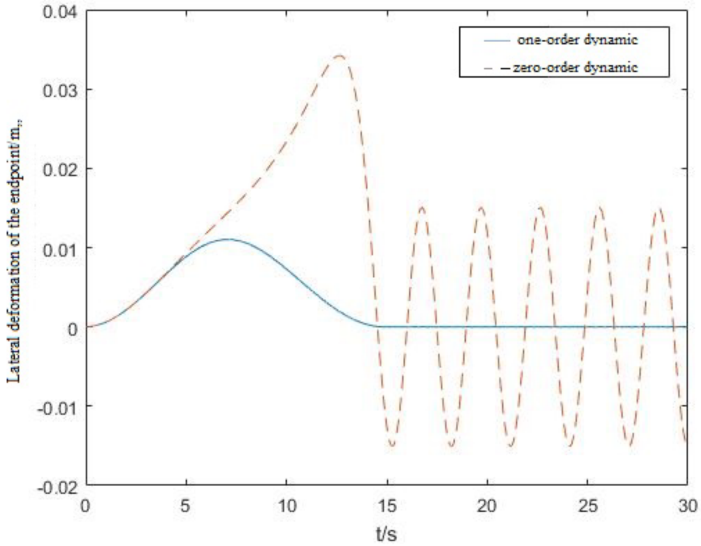

Figure 4 when $\omega = 13.2rad/s$ plate lateral vibration end displacement

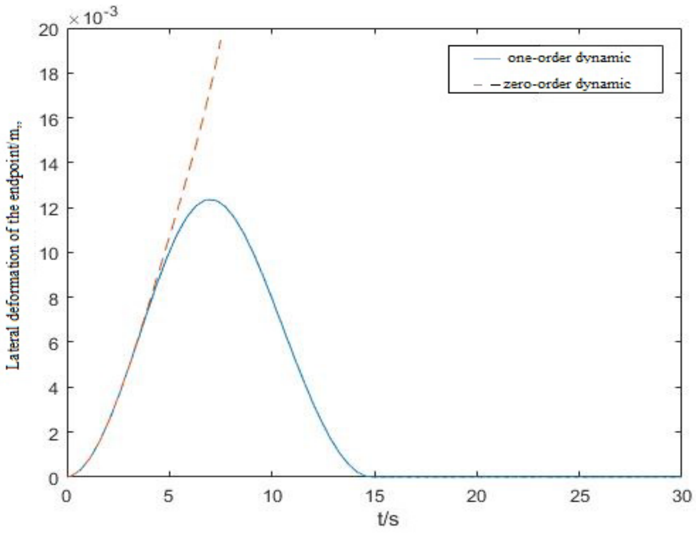

Figure 5 when $\omega = 15rad/s$ plate lateral vibration end displacement

4. Conclusion

In this thesis, the Kane method is used to dynamically model the spacecraft and the flexible attachment. The selection of the coordinate description method is essentially the selection of the generalized rate, which is different from the traditional generalized coordinates, but has a certain correspondence. . Each of the literature on the Kane method indicates how to reasonably select a generalized rate, which needs to be determined based on the experience of the researcher.

The selection of the generalized rate adopted in the paper is the selection method of the reference correlation model. On this basis, the dynamic equation corresponding to the Kane method is derived. The cantilever plate of any motion in space is established and analyzed. In the analysis process, combined with the hypothetical modal method, through the modeling and analysis of the vibration characteristics of the flexible attachment, the simulation results are compared with the simulation results of the lost dynamic stiffness term under the traditional method, and the Kane dynamics model analysis is applied. The necessity of flexibility.

The simulation analysis of the cantilever plate shows that under the simulation analysis result of one model, it is possible to give relatively stable and physics-compliant results for the case of any large-scale motion in space, while the zero-order model that ignores the dynamic stiffness term is small. When the angle is moving slowly, a more stable result can be obtained. However, on the basis of a wide range of fast motion, the result tends to diverge, and the first-order natural frequency is used as the initial state of divergence.

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